The Route Inspection Problem

also known as

The Chinese Postman Problem

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2-nodes, 4-nodes, (and so on) are called *even* nodes or nodes of *even order*.

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Add together the node type numbers of all the nodes in a network. The sum will be twice the number of arcs because each arc is counted twice, once from each end.Thus the sum of node type numbers is even for any network.

An odd number of odd nodes cannot exist because a sum containing an odd number of odd numbers would be odd. Thus any network must have an even number of odd nodes (if it has any odd nodes).

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The problem was first considered by the chinese mathematician *Mei-ko Kwan* in the 1960s, hence its name.

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Therefore it is required that the network is traversable (Eulerian).

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We first deal with traversability (the closed trail), then consider the mimimum weight condition (shortest distance).

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And consider its implications for the postman problem.

If there are no odd nodes in the network, the network is certainly traversable.

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The minimum distance is the sum of the arc distances and any Eulerian trail is an acceptable shortest route, so the problem is trivial.

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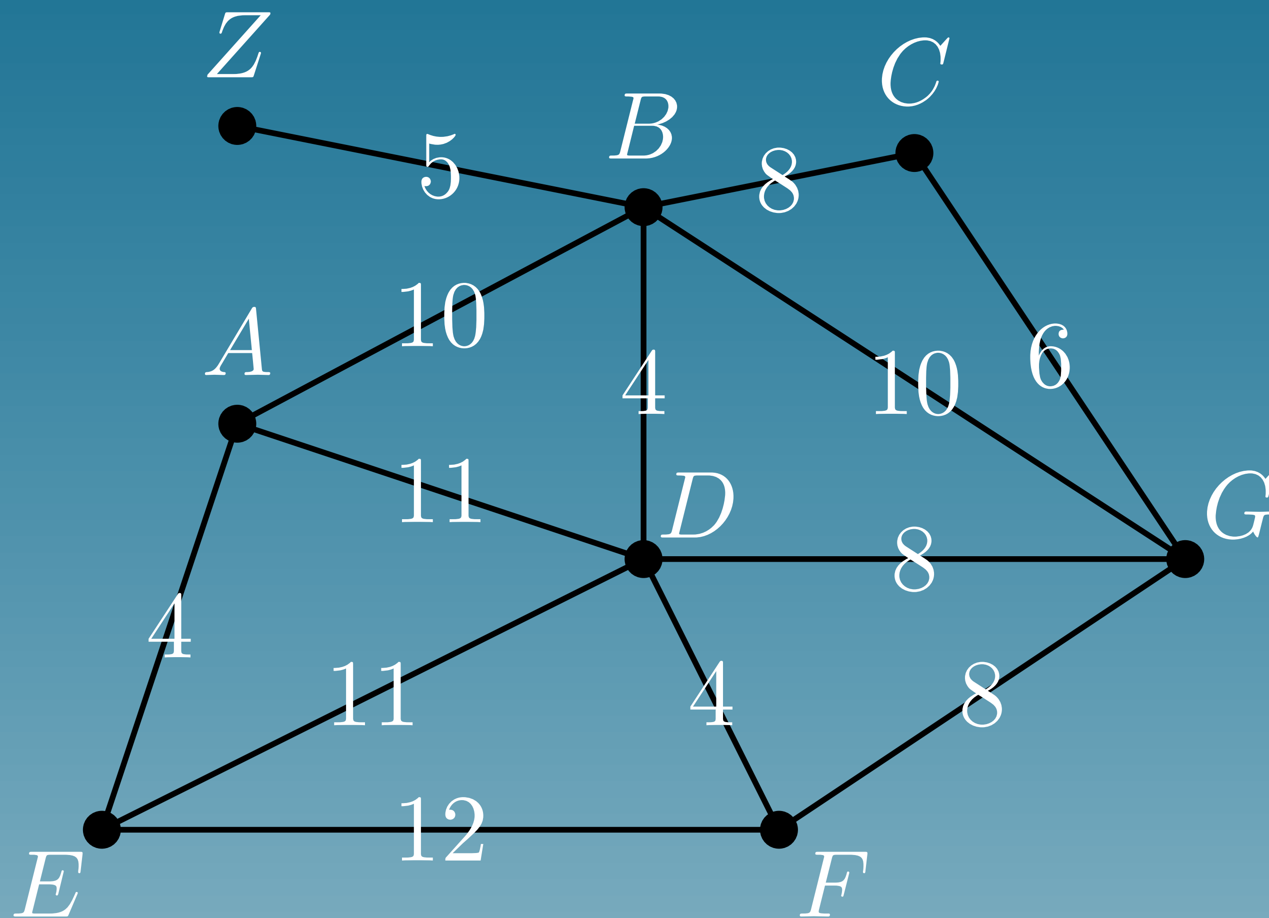
Note that this is always possible because the number of odd nodes is always even.

Example



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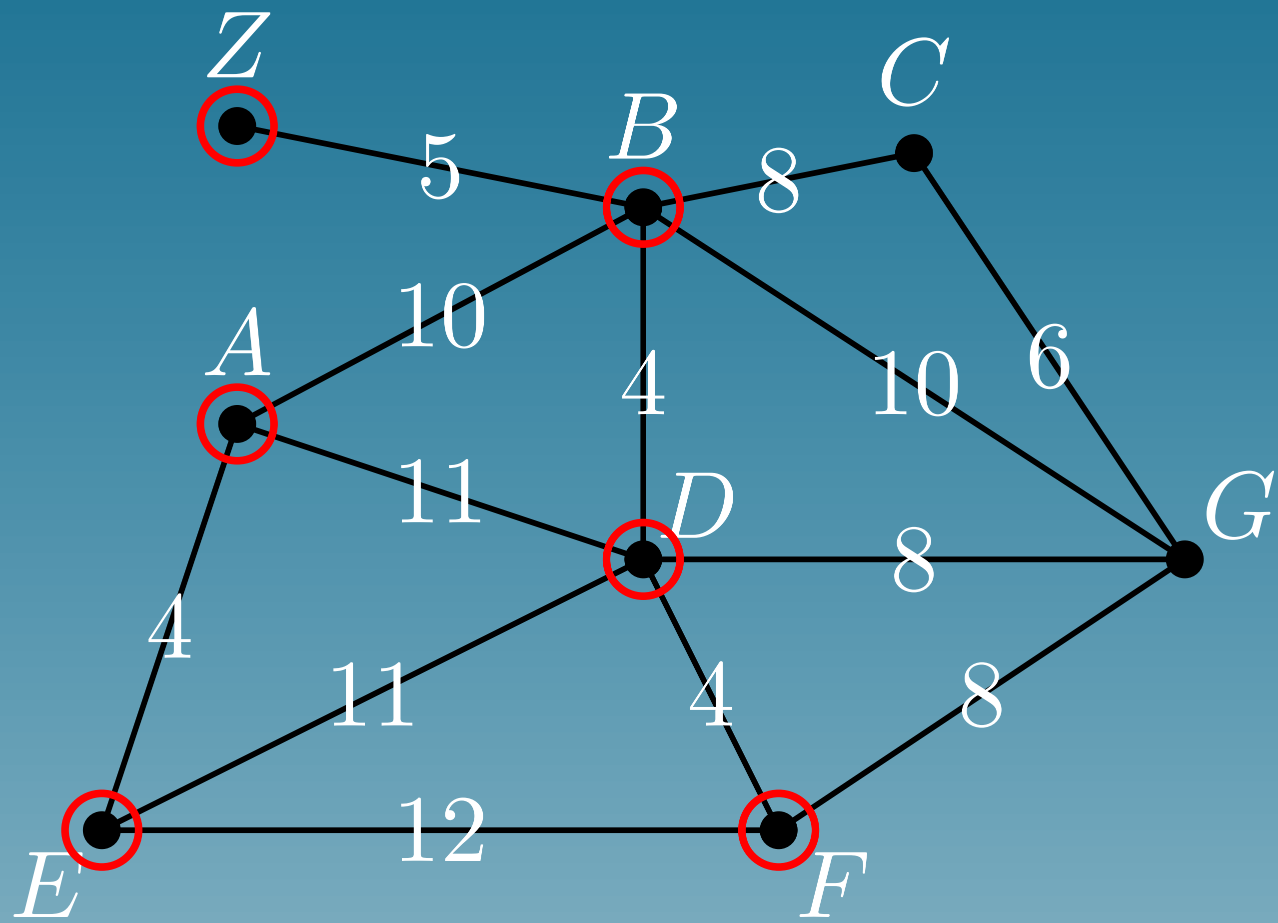

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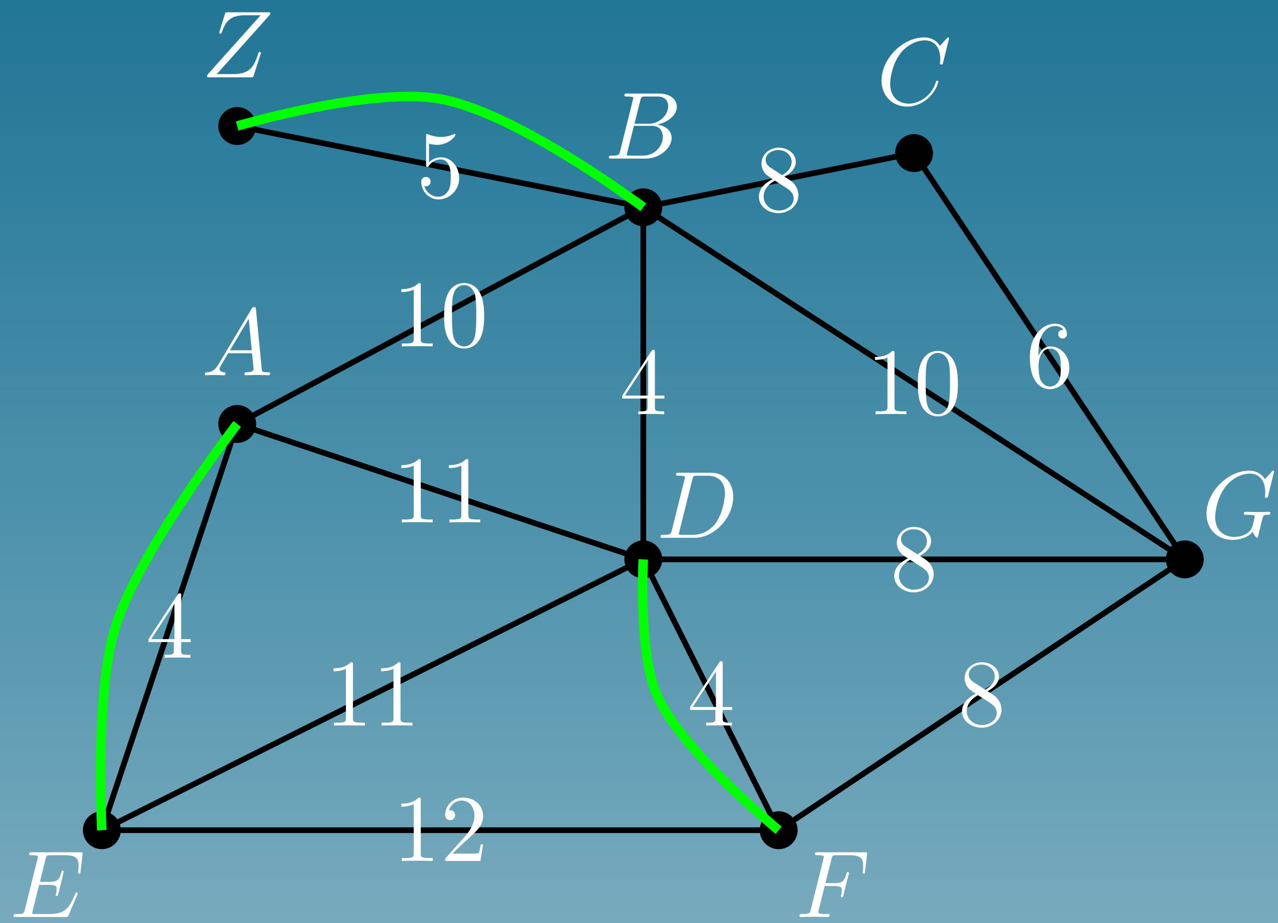

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There are many possible routes around the network. Since all nodes are of even order the postman can start at any node and he finishes there.

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What algorithm can be used to find a shortest path?

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What algorithm can be used to find a shortest path? Dijkstra's algorithm.

It should also be noted that it is not sufficient to only consider one set of pairings of odd nodes, It should also be noted that it is not sufficient to only consider one set of pairings of odd nodes, all such parings must be considered.

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So we have the Chinese Postman Algorithm...

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- Duplicate the minimum weight paths
- Find a trail containing every arc for the new (Eulerian) graph.

- Work through example 5.2.1 (page 59) and example 5.2.2 of OCR D1

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- Practise exercise 5 (page 61) - Yr 12.

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- Practise exercise 5 (page 61) - Yr 12.
- Practise exercise 3C (page 93) - Yr 13.

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A. G. Martin. December 31, 2001